



注意事項：

1. 本科目考試時間共 90 分鐘。
2. 答案卷書寫題號依序作答，不必抄題。
3. 答案卷不可書寫任何可辨別個人姓名或特殊標記，違反者以零分計算。
4. 請於試題紙上填寫准考證號，繳卷時「試題」、「答案卷」一併繳回。

臺中技術學院 95 學年度資訊科技與應用研究所數學科目考題

1. Suppose that the following data represent the invested amount(\$) in new-product development by the manufacturing company over five-year period.

Year x	1	2	3	4	5
Investment, y	1	1	4	4	6

- (1) Find the least squares line for estimating invested amount in new-product development by the company as a function of time.
- (2) Predict the company's investment in new-product development in year $x = 7$.
- (3) What is the estimate of population variance?
- (4) Check the adequacy of the regression model?

Hint: $t_{0.025}(4)=2.77$ $t_{0.05}(4)=2.13$ $t_{0.025}(3)=3.18$ $t_{0.05}(3)=2.35$

2. Suppose the 'Leisure Goods Price Index' of three regions(Taipei, Taichung and Kaohsiung) are given in the following table:

region year	Taipei	Taichung	Kaohsiung
87	105.2	106	105.7
88	108.4	110.7	111
89	113.9	116.9	117.2

Please answer the following questions:

- (1) Is there a significant difference for 'Leisure Goods Price Index' among regions? why or why not?
- (2) Is there a significant difference for 'Leisure Goods Price Index' among years? why or why not?

Hint: $F_{0.05}(3,4)=6.59$ $F_{0.05}(2,4)=6.94$ $F_{0.05}(3,11)=3.59$ $F_{0.05}(2,11)=3.98$

3. To know if products' volume is significantly influenced by machine types. A completely randomized design is performed and data is collected.

	Machine type			
	1	2	3	4
5	5	4	6	3
6	6	9	3	5
4	4	7	4	2
2	2	6	2	7

- (1). To test if the 4 mean volumes are all equal? $\alpha = 0.01$
- (2). To analysis this problem, what assumptions are required.
- (3). What is the estimate of population variance?

Hint: $F_{0.01}(3,15)=5.42$ $F_{0.01}(4,12)=5.41$ $F_{0.01}(3,12)=5.95$ $F_{0.01}(4,15)=4.89$

4. An industrial engineer is interested in the outputs of two assembly lines. Independent tests for the two lines show the following results:

line A	line B
$n_A = 10$	$n_B = 16$
$\bar{x}_A = 14.5$	$\bar{x}_B = 11.3$
$S_A = 0.8$	$S_B = 0.7$

- (1). Construct 95% confidence interval on the true variances for line A and line B respectively.
- (2). Construct 95% confidence interval on the true means for line A and line B respectively.
- (3). Test if variances for line A and line B are equal ($\alpha = .05$), and Using the result to test if mean outputs for line A and line B are equal ($\alpha = .05$).

Hint:

$\chi^2_{0.975}(10) = 20.48$	$\chi^2_{0.975}(9) = 19.02$	$\chi^2_{0.95}(10) = 18.31$	$\chi^2_{0.95}(9) = 16.92$
$\chi^2_{0.025}(10) = 3.25$	$\chi^2_{0.025}(9) = 2.7$	$\chi^2_{0.05}(10) = 3.94$	$\chi^2_{0.05}(9) = 3.33$
$\chi^2_{0.975}(16) = 28.85$	$\chi^2_{0.975}(15) = 27.49$	$\chi^2_{0.95}(16) = 26.30$	$\chi^2_{0.95}(15) = 25$
$\chi^2_{0.025}(16) = 6.91$	$\chi^2_{0.025}(15) = 6.26$	$\chi^2_{0.05}(16) = 7.96$	$\chi^2_{0.05}(15) = 7.26$
$t_{0.05}(10) = 1.81$	$t_{0.025}(10) = 2.23$	$t_{0.05}(9) = 1.83$	$t_{0.025}(9) = 2.26$
$t_{0.05}(16) = 1.75$	$t_{0.025}(16) = 2.12$	$t_{0.05}(15) = 1.75$	$t_{0.025}(15) = 2.13$
$t_{0.025}(24) = 2.06$	$t_{0.05}(24) = 1.71$		
$F_{0.975}(9,15) = 3.12$	$F_{0.95}(9,15) = 2.59$	$F_{0.975}(10,16) = 2.99$	$F_{0.95}(10,16) = 2.49$

5. Determine the number of positive integer solutions for $x_1 + x_2 + x_3 + x_4 + x_5 < 21$, where $x_i \geq 1$.

6. Use mathematical induction to prove that $2^n \times 2^n - 1$ is divisible by 3 for $n \in \mathbb{N}$.

7. Give an example to illustrate the Kruskal's algorithm for the minimum spanning tree.

8. Find parametric equations of the line through $p_0(3,2,4)$ parallel to $\mathbf{v} = \begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix}$.

9. Solve the following linear systems by Cramer's rule:

$$\begin{aligned} -2x_1 + 3x_2 - x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 4 \\ -2x_1 - x_2 + x_3 &= -3 \end{aligned}$$

10. Let $A = \begin{bmatrix} 3 & -5 \\ 1 & -3 \end{bmatrix}$. Compute A^9 . (Hint: Find a matrix P such that $P^{-1}AP$ is a diagonal matrix D and show that $A^9 = PD^9P^{-1}$.)