NTIT 344國立台中技術學院	所別：資訊科技與應用研究所	准考證號碼：
九十三學年度研究所碩士班入學考試試題	科目：計算機基礎數學	考生姓名：

注意事項：

1.本科目考試時間共 90 分鐘。

2.答案卷書寫題號依序作答，不必抄題。

3.試卷不可書寫任何辨別個人姓名或特殊標記，違反者以零分計算。

4.請於試題簽名並填寫准考證號碼，繳卷時「試題」、「答案卷」一併繳回。

1. Prove by induction that for $n \geq 1$,
$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! = (n+1)! - 1$
where $n!$ stands for the product $1 \cdot 2 \cdot 3 \cdot \cdot \cdot n$ (10%)

2. Let A = {a, b, c, d, e, f, g, h, i, j, k}, π_1 = {{a, b, c, d}, {e, f, g}, {h, i}, {j, k}}, π_2 = {{a, b, c, h}, {d, i}, {e, f, j, k}, {g}} be two partitions on set A, find (a) $\pi 1 \cdot \pi 2$ and (b) $\pi 1 + \pi 2$. (10%)

3. From the integers of the set {1, 2, 3, …, 400}, 201 of them are chosen arbitrarily. Show that, among the chosen numbers, there exist two such that one divides another. (10%)

4. Solve the recurrence relation: $f(n) = 4 f(n/2) + 2 n^2$, where n is a power of 2, and f(1) = 1. (10%)

5. Give an example to illustrate the Kruskal's algorithm for the minimum spanning tree. (10%)

6. Find a Confidence Interval for the $\mu_1 - \mu_2$ between two independent Normal distributions, if the σ_1, σ_2 are known, but not necessarily equal. (10%)

7. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(0, \theta)$, show that $\sum X_i^2 / n$ is an unbiased estimator of θ. (20%)

8. In order to test at the α=0.05 significant level, the hypothesis H0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ against all possible alternative hypothesis. The sample sizes n=12.

$$\sum \sum (X_{ij} - \bar{\bar{X}})^2 = 80, \quad \sum \sum (\bar{X}_i - \bar{\bar{X}})^2 = 30$$

(1) Please complete the following ANOVA table.
(2) Determine whether we accept or reject H_0 ? (20%)

ANOVA Table

Source	Sum of Squares	Degree of Freedom	Mean Square	F-ratio
Treatment				
Error				
Total				

10.Giving n=11 observations of X and m=13 observations of Y, where X is $N(\mu_{x,} \sigma_x)$ and Y is $N(\mu_{y,} \sigma_y)$. Using the samples we obtain $\overline{X}=1.03$, $S_x^2 = 0.24$, $\overline{Y}=1.66$, $S_Y^2 = 0.35$. (30%)

(1) Determine whether we accept or reject H_0 , When

H_0 : $\sigma_x^2=\sigma_y^2$ H_1 : $\sigma_x^2 \neq \sigma_y^2$

(2) Determine whether we accept or reject H_0 , When

H_0 : $\mu_x=\mu_y$ H_1 : $\mu_x^2 < \mu_y^2$

Reference data : $F_{0.95}(4,8)=3.84$ $F_{0.95}(3,8)=4.07$ $F_{0.95}(8,4)=6.04$

$F_{0.95}(8,3)=8.85$ $F_{0.975}(10,12)=3.37$ $F_{0.95}(10,12)=2.75$

$F_{0.975}(12,10)=3.62$ $F_{0.95}(12,10)=2.91$

$t_{0.95}(22)=1.717$ $t_{0.975}(22)=2.074$ $t_{0.95}(23)=1.714$

$t_{0.975}(23)=2.069$