



注意事項：

1. 答案依序書寫於答案卷上，不必抄題。
2. 答案卷不可書寫任何可辨別個人姓名或特殊標記，違者不予計算。
3. 請於試題紙上填寫准考證號碼，繳卷時「試題」、「答案卷」一併繳回。

(1). 何謂『中央極限定理』？(請儘量以數學式說明之。)(10%)

(2). 抽樣調查台中市 1000 戶，結果有 250 戶空屋，(15%)

1. 求台中市空屋率 95% 信賴區間。
2. 是否有顯著的證據說台中市空屋率超過 20%，為什麼？($\alpha = 0.05$)

(3). 若 X 、 Y 是獨立的波瓦松分配，而其分配函數如下：(15%)

$$f(x) = \frac{m^x e^{-m}}{x!}, x = 0, 1, 2, \dots, f(y) = \frac{n^y e^{-n}}{y!}, y = 0, 1, 2, \dots$$

試求：

1. $X + Y$ 之分配。
2. $f(x | x + y = 50)$.

(4). Let X, Y be two random variables and if X has a Bernoulli distribution with parameter p (that is, $P(X = 1) = p = 1 - P(X = 0)$), and $E(Y|X = 0) = 1$, and $E(Y|X = 1) = 2$, what is $E(Y)$? (10%)

(5). A coin is tossed four times. Let X denote the number of times a head is followed immediately by a tail. Find the distribution, mean, and variance of X . (15%)

(6). Suppose that the random variable X takes on the values: $0, 1, \dots$ with the following probabilities: (15%)

$$f(j) = P(X = j) = \frac{c}{3^j}, j = 0, 1, \dots$$

1. Determine the constant c and compute the following probabilities:
2. $P(X \geq 10)$;
3. $P(X \in A)$, where $A = \{j; j = 2k + 1, k = 0, 1, \dots\}$

(7). A regression analysis from a sample of 15 produced the following: (20%)

$$\sum(x_i - \bar{x})(y_i - \bar{y}) = 160, \sum(x_i - \bar{x})^2 = 180, \sum(y_i - \bar{y})^2 = 190, \sum(y_i - \hat{y})^2 = 40, \bar{x} = 12, \bar{y} = 56$$

1. Find the estimated regression model (Sample model) $\hat{y}_i = b_0 + b_1 x_i$, that is b_1 and b_0 .
2. Find the estimate of the standard error for the model, that is s_e .
3. Find the estimate for the standard deviation of the slope, that is s_{b_1} .
4. Provide a 95% prediction confidence interval for $E(y|x_p = 16)$.

$$\text{where } t_{0.05,15} = 1.7531, t_{0.025,15} = 2.1315, t_{0.05,13} = 1.7709, t_{0.025,13} = 2.1604$$